# Chapter Seven

## Techniques of Integration

Integration by Parts:

$$\int u\,dv = uv - \int v\,du$$

If $f(x) = p(x)\cdot q(x)dx$ with $p$ polynomial and $q$ transcendental, and if $q$ = *exp* or *trig* then let $u = p$ and $dv = qdx$, but if $q$ = *log* or *inverse trig* then let $u = q$ and $dv = pdx$.

Trigonometric Identities:

$$\sin^2 x + \cos^2 x = 1 \qquad \tan^2 x + 1 = \sec^2 x \qquad \tan x = \frac{\sin x}{\cos x}\ ,\ \sec x = \frac{1}{\cos x}$$

Angle Sums and Differences:
$$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$$
$$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$$

Double-Angle Identities:
$$\sin(2A) = 2\sin A\cos A$$
$$\cos(2A) = \cos^2 A - \sin^2 A \ = \ 2\cos^2 A - 1 \ = \ 1 - 2\sin^2 A$$

Half-Angle Identities:
$$\cos^2 A = \tfrac{1}{2}(1 + \cos 2A)\ ,\ \sin^2 A = \tfrac{1}{2}(1 - \cos 2A)$$

Products to Sums:
$$\cos A\cos B = \tfrac{1}{2}[\cos(A - B) + \cos(A + B)]$$
$$\sin A\sin B = \tfrac{1}{2}[\cos(A - B) - \cos(A + B)]$$
$$\sin A\cos B = \tfrac{1}{2}[\sin(A - B) + \sin(A + B)]$$

Even/Odd:
$$\cos(-x) = \cos(x)\ ,\ \sin(-x) = -\sin x$$

Trig Substitutions:

$f(x^2 + a^2) \Rightarrow x = a\tan\theta\ ,\ dx = a\sec^2\theta d\theta\ ,\ x^2 + a^2 = a^2\sec^2\theta$

$f(x^2 - a^2) \Rightarrow x = a\sec\theta\ ,\ dx = a\sec\theta\tan\theta d\theta\ ,\ x^2 - a^2 = a^2\tan^2\theta$

$f(a^2 - x^2) \Rightarrow x = a\sin\theta\ ,\ dx = a\cos\theta d\theta\ ,\ a^2 - x^2 = a^2\cos^2\theta$