

Math 7411 Homework Assignment 1 Spring 2008

Due January 24.

1. Let $A_{i,j}$ be subsets of a set X for $i, j \in \mathbb{N}$. Show that

$$\bigcap_{i=0}^{\infty} \bigcup_{j=0}^{\infty} A_{i,j} = \bigcup_{(a_i)} \bigcap_{i=0}^{\infty} A_{i,a_i}$$

where the second union is over all sequences $(a_i)_{i=0}^{\infty}$ of natural numbers.

2. Give an example of a partial order with a unique minimal element, but no smallest element.
3. Show that there exists a function $f: \mathbb{R} \rightarrow \mathbb{R}$ such that $f(x + y) = f(x) + f(y)$, but $f(x)$ is *not* of the form $f(x) = \lambda x$. [Hint: Consider \mathbb{R} as a vector space over the field \mathbb{Q} and define a suitable f using a basis for this vector space.]
4. Using Zorn's Lemma, prove Hausdorff's Maximal Principle.
5. Using Zorn's Lemma, prove that for any two sets A and B , either there is an injective function $f: A \rightarrow B$, or there is an injective function $g: B \rightarrow A$. [Hint: Consider the set of injective functions $f': A' \rightarrow B$, $A' \subseteq A$.]