4614/6614 **Test 2 Examples** **Spring 2014**

1. Let the random variable X have p.d.f. $f(x) = cx^2 + x$ for $0 \leq x \leq 2$; and $f(x) = 0$ otherwise.
   (a) Find the value of c that makes this a valid p.d.f.
   (b) Find the cumulative distribution function $F(x)$.
   (c) Find $F(-1)$, $F(0)$, $F(1)$.
   (d) Find $\mathbb{P}(0 \leq X \leq 0.5)$.
   (e) Find $\mathbb{E}(X)$ and $\text{Var}(X)$.

2. The m.g.f. of X is given by $M_X(t) = 1/(1-5t)^2$.
   (a) Find $\mathbb{E}(X)$.
   (b) Find $\text{Var}(X)$.

3. Random variable X and Y have joint p.m.f. given by this table

$Y \backslash X$	3	4	5
1	$\frac{1}{6}$	$\frac{1}{6}$	0
2	$\frac{1}{6}$	0	$\frac{1}{6}$
3	0	$\frac{1}{6}$	$\frac{1}{6}$

   (a) Find the marginals of X and Y.
   (b) Are X and Y independent?
   (c) Find the correlation coefficient ρ between X and Y.

4. Random variables X and Y have joint p.d.f. given by $f(x, y) = e^{-x-y}$, $x, y \geq 0$. Are X and Y independent?

5. The conditional p.d.f. of X given Y is $f_{X|Y}(x|y) = ax$ for $1 < x < y$; and 0 otherwise. The marginal p.d.f. of Y is $f_Y(y) = \frac{b}{y^3}$ for $y > 1$; and 0 otherwise.
   (a) Find a and b. (Note that a may depend on y.)
   (b) Find the joint p.d.f. of X and Y.
   (c) Find $\mathbb{P}(X < 2 \mid Y = 3)$.

6. A machine operation produces steel shafts where diameters have a normal distribution with mean equal to 1.005 inches and a standard deviation of 0.01 inches. Specifications call for diameters to fall within the interval 1.00 ± 0.02 inches. What percentage of the output of this machine operation will fail to meet specifications?