4614/6614 **Test 3 Examples** **Spring 2014**

1. Let $X_1, X_2, \ldots, X_n$ be a random sample from a distribution with p.d.f. $f(x) = \frac{2\theta^2}{x^3}$, $x > \theta$.

   (a) Find a method of moment estimator for θ.

   (b) Find a maximum likelihood estimator for θ.

2. Let $X_1, X_2, \ldots, X_n$ be a random sample from a distribution with p.d.f. $f(x) = \theta(1-\theta)^x$, $x = 0, 1, 2, \ldots$ Find the maximum likelihood estimate of θ.

3. Let $X_1, X_2, \ldots, X_n$ be a random sample from a distribution with mean $\mathbb{E}(X) = \mu$ and variance, $\mathrm{Var}(X) = 1$. Let an estimator for μ^2 be

$$\hat{\mu^2} = \overline{X}^2.$$

   Show that $\hat{\mu^2}$ is a biased and find the bias $\mathbb{E}(\hat{\mu^2}) - \mu^2$.

4. Find a 98% confidence interval for the mean of a normal distribution with variance 4 from the following sample:

   1.2 3.7 2.1 5.7 4.1 7.2

5. A consumer group conducted a study to determine the reliability of a certain brand of compact disk player. A random sample of ten players gave the following results in years of service:

   4.4 5.3 6.2 8.0 4.0 3.9 7.7 5.2 6.8 6.6

   (a) Find a 99% confidence interval for the mean lifetime of the players.

   (b) Find a 95% confidence interval for the variance of the lifetime of the players.

6. It has been suggested that recent college graduates spend more time watching television to obtain news information than reading newspapers. To check this a sample of graduates was taken and the following data was collected on the number of hours per week they spent watching television news. The data are summarized below. Assuming independent normal distributions and equal variances, construct a 90% confidence interval for the difference in the number of hours per week for the two groups. Interpret your results (i.e., is there strong evidence for a difference?).

Group	Number	Mean	S
Less than 5 years	12	15.6	2.3
More than 10 years	11	10.1	2.2

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1. Suppose one needs to flip a fair coin N times to obtain a total of 50 heads. Estimate $\mathbb{P}(N \geq 110)$ by using a normal approximation for N. [You may assume that the number of flips necessary to obtain one head is geometric with mean 2 and variance 2.]

2. For each of the following situations, describe suitable null and alternative hypotheses and describe what type I and type II errors correspond to.

   (a) Last year, the mean contribution to the United Way charities in a certain city was 200 dollars. A reporter for Philanthropy Today magazine feels that it has changed since then.

   (b) According to the Department of Transportation, last year the mean price of a gallon of regular gasoline was 2.50, the department feels that the mean price has increased since then.

3. There are two boxes. Box I contains 5 red balls and 5 black balls. Box II contains 2 red balls and 8 black balls. A box is selected at random and 2 balls removed without replacement. It is agreed to conclude that Box II was selected if both balls are black. The null hypothesis is that Box I was selected, and the alternative hypothesis is that Box II was selected. Calculate $\alpha = \mathbb{P}(\text{Reject } H_0 \mid H_0)$ and $\beta = \mathbb{P}(\text{Accept } H_0 \mid H_1)$.

4. A random sample of size $n = 25$ is taken from a normal population with $\sigma = 6$. The sample mean was found to be $\overline{X} = 44$. For $\alpha = 0.01$, text $H_0 : \mu = 40$ versus $H_1 : \mu > 40$.

5. To compare the durabilities of two patching materials for highway use, patches were laid down on an interstate highway at six locations. After a one month trial, wear indicators were examined (the higher the reading the more wear the patch exhibits). Test whether there is a difference in mean wear between the two materials at a 90% significance level. [You may assume that the wear is normally distributed.]

Location	1	2	3	4	5	6
Material A	11.7	12.5	11.3	8.7	9.4	10.3
Material B	11.6	10.5	9.4	7.4	9.7	9.3