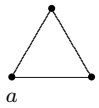
7281 Test Example Questions Fall 2007

1. Find the volume of the square-based pyramid with base $(1, 2, 0)$, $(2, 3, 0)$, $(0, 2, 1)$, $(1, 3, 1)$, and apex $(2, 0, 5)$.

2. Suppose A and B are 3×3 matrices. Evaluate the following determinants in terms of $\det A$ and $\det B$.
(i) $\det(A^{-1}B)$ (ii) $\det(A^T A)$, (iii) $\det(2A)$, (iv) $\det(\operatorname{adj}(A))$.
(Recall $A \operatorname{adj}(A) = (\det A)I$.)

3. If it is rains today, then with probability 0.6 it will rain tomorrow, but if it does not rain today, then with probability 0.2 it will rain tomorrow. On average, on what proportion of days in the long run does it rain?

4. In the following graph, how many walks of length 8 are there from a to a.



5. Solve the second order differential equation

$$x'' - 5x' + 6x = 0$$

by first writing it as a system of two first order equations

$$x' = y, \qquad y' = 5y - 6x.$$

6. Suppose A is a diagonalizable (real) square matrix with all eigenvalues non-negative. Show that A has a real "square root", i.e., there is a real matrix B with $B^2 = A$.

7. Let A be any real matrix. Show that $A^T A$ is symmetric and that all the eigenvalues of $A^T A$ are non-negative.

8. Let W be a subspace of $\mathbb{R}^4$ spanned by the vectors $[2, 1, -2, 4]$ and $[5, 3, 2, -1]$. Find an orthonormal basis for W.

9. Find a basis for the orthogonal complement of the subspace W of $\mathbb{R}^4$ spanned by the vectors $[2, 1, -2, 4]$ and $[5, 3, 2, -1]$.

10. Suppose that A is an orthogonal matrix. Evaluate $A(A^T)^{-1}A^{-1}A^T$.